only about one-third that suggested by Prof. Langley. With the view of obtaining a more correct determination, Prof. Müller has proposed, in connection with Dr. Kempf, to observe the brilliancy of a star simultaneously, as seen at sea-level and on the top of a neighbouring mountain, thus getting different lengths of atmospheric path; but the few observations that have hitherto been made with this object do not point to any trustworthy conclusion.

In the section devoted to the description of photometric apparatus, one looks for completeness rather than originality, and though Dr. Müller may have omitted some of the forms that have been recommended, yet so many examples have been illustrated and described, one would willingly hope that he has exhausted the entire catalogue. Practically there are only two principles which underlie every form of photometric measurement—extinction, and equalisation of light—but ingenuity has applied these two principles in every variety of method. This variety suggests great dissatisfaction with existing apparatus, and insinuates that a simple, convenient photometer, that would inspire confidence and ensure accuracy, has yet to be invented. After all, the eye is the real photometer; and it may be that here the inherent difficulty, physiological rather than mechanical, exists, though attempts to displace the eye, and to substitute the photographic film, have not as yet led to more satisfactory results. We cannot follow Dr. Müller through the various arrangements of diaphragms and the employment of absorbing media, by which the problem has been sought to be solved on the principle of extinction. Neither can we mention the many advocates of the method of equalisation, who have pressed into the service of photometry rotating discs, reflecting spheres, and other ingenious devices, while the applications of polarised light in various forms are legion. The special department of spectral photometry receives new additions to its apparatus every month; but if any one imagines that he has a new scheme to add to the many already proposed, we advise him to study the critical remarks of our author before venturing on a ground which is already strewn with failures, or, at least, doubtful successes.

The third and concluding section is not the least interesting. Here it is the author's purpose to show what has been accomplished by the application of photometrical methods to the planets and satellites, to the comets and stars. We not only get an historical account of what has been effected, and are thus reminded of the energy that astronomers of this century mainly have devoted to this subject, but are able to appreciate the degree of success that has attended their efforts, and see in what direction future enterprise is possible. From an observer of Dr. Müller's experience we get many critical remarks of great value, and are able to see the exact position in which celestial photometry stands. Perhaps, in the single direction of the application of photography, something more might have been added; but it must be remembered that the author's colleague, Dr. Scheiner, has already fairly occupied this ground, and some large catalogues, like the "Southern Durchmusterung" of Dr. Kapteyn, had not appeared at the time of publication. In this section, where so much is important, it is difficult to select any one topic for comment; but special mention should be made of the references to the observation of variable stars, and the relations existing between eye-estimations of star magnitude and photometric measurement. With this last subject is necessarily connected the difficult question of standards and units of measurement. Over such subjects, to which may be added the desirability of attaining uniformity of scale, some little controversy has hitherto ruled, and perhaps is not yet definitely settled, owing to the diversity in the instruments employed, and the systematic errors introduced, it may be, by abnormal vision. On the whole, Dr. Müller preserves a judicial dignity and impartiality, but some of the strictures concerning the Southern Catalogues produced under Harvard methods and auspices are, we think, undeserved. The lower altitude of the Pole at Arequipa did undoubtedly give rise to great difficulties, but any want of continuity, supposing it to exist, between the Northern and Southern Catalogues can hardly be ascribed to the method of observation. Dr. Müller's own method of establishing numerous points of reference, scattered over the sky, to serve as standard comparison stars, has much to recommend it, but it is a plan that involves long preparation before the work of real cataloguing can begin. How far the increased accuracy of the work repays for this preliminary labour, and to what extent other astronomers will avail themselves of these scattered standards, rather than to trust to a single star, are points that can hardly be decided yet; but every attempt that aims at additional accuracy must be welcomed, and on the side of accuracy we believe that the Potsdam photometric measures stand unrivalled.

AN ILLUSTRATED HISTORY OF MATHEMATICS.

Histoire des Mathématiques. Par J. Boyer. Pp. xii + 260. (Paris: G. Carré et C. Naud, 1900.)

TO write within the compass of less than 240 octavo pages an intelligible outline of the history of mathematics from the earliest times to the present day is a task so difficult that its accomplishment might reasonably be thought impossible. Yet by adopting a consistent, if modest, programme, and frankly accepting its necessary limitations, M. Boyer has achieved a considerable measure of success; possibly as much as the scope of his essay could allow. A reader whose mathematical knowledge is only moderate, or even elementary, will obtain from this book some idea, trustworthy so far as it goes, of the contributions made to the science of mathematics by the ancient Babylonians, Egyptians, Indians and Greeks; he will be able to follow the feeble course of geometry and analysis through the Middle Ages, and appreciate to some extent the glorious renascence of the sixteenth and seventeenth centuries; and he will acquire some conception of the work of the giants, from Newton to Laplace, who established the primary landmarks of modern mathematics.

Perhaps the most satisfactory chapters in the book are xiv.-xvii., which cover the period from Descartes to Laplace. It deserves to be specially remarked that M. Boyer discusses the invention of the infinitesimal calculus in an impartial and charitable spirit, and that he does substantial justice to the merits of Cauchy and Monge, who are not always appreciated as they deserve, even by their own countrymen.

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As might be expected, the chapter on contemporary mathematics and mathematicians is meagre, and lacking in proportion; but it has the merit of emphasising some, at least, of the most important lines of inquiry, and it is not disfigured by any of those erroneous statements which so often appear in summaries of this kind.

One feature of the book, which will make it attractive to all classes of readers, consists in the illustrations. They comprise four facsimiles of manuscripts, one of the title-page of the "Acta Eruditorum," two plates showing the form of mathematical instruments in the seventeenth century, and nineteen portraits. The portraits are fairly representative, and are derived from authentic originals; one would gladly have dispensed with Mme. du Chatelet and Saunderson in exchange for Gauss and Abel, who are unaccountably omitted.

The appearance of so many popular histories of mathematics lately suggests a few remarks upon the purpose which they are, or should be, designed to fulfil. Their proper object is that of supplying stimulus; and there is no doubt that the interest of a mathematical student is greatly encouraged by historical notes on the subject of his reading. To the teacher, the history of mathematics is a subject not merely of interest, but of vital importance, because the psychological history of a race tends to repeat itself, in little, in the mind of the individual; hence the proper order of teaching a subject is not necessarily the most logical one; and indeed the best compromise between logical sequence and what we may call "historical sequence" is precisely the golden mean at which the teacher must do his best to aim. A popular history does good if it awakens the teacher to some idea of this: the great risk is that he may imagine that the popular history contains all the information that he requires. This is so far from being the case that he may be actually in a worse position after reading his "History" than before. Points of the highest importance are necessarily ignored in a popular treatise, either from want of space, or because the author is afraid of frightening his readers with technicalities. For instance, it is comparatively easy to ascertain the net results of Greek geometry, expressed in modern terminology; but without some acquaintance with the actual works of Euclid, Archimedes and Apollonius, *as they wrote them*, it is simply impossible to have any correct ideas of the aims and methods of Greek mathematicians. If our teachers of mathematics were really familiar even with the "Elements" of Euclid, instead of with a garbled version of a part of it, they would be far better able to discuss intelligently the question of "Euclid and his Modern Rivals."

Again, the real interest of the ante-Descartes period in Europe consists in the gradual improvement of algebraic notation, and of methods of arithmetical computation. This question of notation is of the greatest interest from every point of view. What we have now is simply the survival of the fittest, and may have to submit to modifications more drastic than any of us at present imagine; still it is far ahead of its predecessors, and our admiration of Archimedes and Fermat is greatly enhanced when we realise the wretchedly inadequate notation which they had to employ. To have traced, even in a general way, this advance of notation and method, is far more instructive and important than to know, for instance, that logarithms were invented by Napier of Merchiston, or that Newton discovered the Binomial Theorem; yet very little help in this direction is afforded by the popular history.

A really good history of mathematics in the nineteenth century has yet to be written; it would probably require the combined labour of an organised body of experts, such as those engaged on that invaluable work, the "Encyclopädie der mathematische Wissenschaften." Until such a scientific history has been composed, it is idle to expect anything worth reading in a popular treatise. It may even be questioned whether a popular writer, however competent, could profitably deal with the subject at all, unless our methods of school teaching are greatly modified. For the history of modern mathematics is not mainly that of individual discoveries, however brilliant; but that of the systematic investigation of mathematical notions, such as "number," "continuity," "function," "limit," and the like. If these technical terms are ignored, even a popular sketch of the subject becomes impossible; yet how many of these terms are even approximately understood by any but mathematical specialists? And what is the use of trying to explain the theory of doubly periodic functions to readers who are unaware that, in learning trigonometry, they were studying singly periodic functions without knowing it?

G. B. M.

SCIENTIFIC LENS-MAKING.

Theorie und Geschichte des photographischen Objectivs. Von Moritz von Rohr. Pp. xx + 435. (Berlin: Springer, 1899.)

DR. VON ROHR'S book contains much to attract students interested in the theory and development of photographic lenses. The author is one of the scientific workers attached to Zeiss' manufactory in Jena, and has a practical acquaintance with his subject.

The book is most instructive to an English reader, specially for the reason that while the debt due to the great English opticians of the present day, as well as to those of the past, is freely owned, and the author's appreciation of the value of their work is warm and cordial, yet the contrast between the methods of the English school and those of the pupils of Abbé and Schott is sharply drawn, and it is clear that in the opinion of the author the future is with the latter. Schott and Abbé began in 1881 "to study carefully, as far as possible, all chemical elements, which in any form can become constituents of amorphous compounds produced by fusion, with regard to their influence on the refraction and dispersion of the compounds." On this secure basis is raised the great Jena glass factory, to whose work the scientific world is already so deeply indebted.

We doubt if any English manufacturer would have attempted thus to improve his products; the new methods, the methods which Englishmen must adopt if England is to retain her place among the manufacturing peoples of the world, have not yet found a home among us, and unless a change is made, England must cede the place of honour, not merely in lens-making, but in every branch of manufacture.

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